ST. HENRY'S COLLEGE KITOVU

A'LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2019

STATISTICS AND PROBABILITY

1. (a) The data below was obtained from a survey carried out on the temperature variations of 10 cities in a cold season of the year.

0^oC, -2^oC, 3^oC, 4^oC, -4^oC, -10^oC, 5^oC, 1^oC, -8^oC, -7^oC.

Determine the;

(i) mean temperature,

(ii) variance.

(b) A sample of **n** members of the rotary club of Masaka was asked how many crates of beer they took in a given month.
The results were as follows $\sum \boldsymbol{x} = \mathbf{225}, \sum \boldsymbol{x^2} = \mathbf{1755}$. Find the possible values of **n** if the standard deviation is **1.5** and hence find the respective mean number of crates taken by the Rotarians.

2. The table shows the marks scored by a group of candidates in a mathematics exam.

Marks (%)	10 - 19	20 - 29	30 - 34	35- 44	45-54	55-64	65-69
Frequency density	0.7	2.6	4.2	3.8	4.6	2.8	2.6

(a) Draw a histogram and use it to estimate the modal mark.

(b) Calculate the;

(i) mean mark,

(ii) median mark,

(iii) standard deviation.

3. The table below shows the marks obtained by a group of students in a math test.

Marks(%)	$20-29$	$30-34$	$35-44$	$45-64$	$65-74$	$75-84$
Frequency	5	5	12	20	10	8

Calculate the;

(a) mean mark,

(b) standard deviation,

(c) mode,

(d) median,

(e) semi-interquartile range,

(f) middle 60% of the marks.

(g) number of students whose marks exceed 47%.

4. The table below shows the heights (y) in centimetres and the age (x), in years, of a group of students in a certain secondary school.

Student	A	B	C	D	E	F	G	H	I	J	K	L
Age (x)	12	14	13	15	17	20	17	15	18	19	14	16
Height (y)	130	136	120	120	153	160	155	142	145	172	140	157

(a) Construct a scatter diagram, draw the line of best fit and comment hence estimate x when y = 142.

(b) Giving rank 1 to the tallest student and oldest student, calculate the rank correlation coefficient and comment at 5% level of significance.

5. A class of 10 students were examined in Economics (E) and Mathematics (M). The following table shows their scores out 10.

Student	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
E	7.2	5.2	3.1	3.8	8.1	5.2	4.0	6.0	6.3	7.5
M	6.4	6.0	6.5	4.3	7.0	4.8	3.4	6.2	5.9	6.0

(a) Plot the above on a scatter diagram.

(b) Draw a line of best fit and estimate the value of E when M = 8.6.

(c) Compute the rank correlation coefficient between E and M and comment on you result.

6. (a) The table below shows the expenditure (in Ug. shs) of a student during the first and second terms.

ITEM	EXPENDITURE		WEIGHT
	First term	Second term	
Clothing	46,500	49,350	5
Pocket money	55,200	57,500	3
Books	80,000	97,500	8

Using first term expenditure as the base, calculate the average weighted price index to **one** decimal place.

(b) The price relative of commodity in 2010, using 2009 as base year was 105. The price relative of same commodity in 2012 using 2010 as base year was 95. Given that the cost of the commodity in 2009 was shs. 259,250, find its cost in 2012.

(c) 2015 being the base year, the price index of a particular commodity in 2017 was 110 and if 2017 is used as the base year, the price index in 2018 is 120. Calculate the index number for 2018 taking 2015 as the base year.

7. (a) Two events A and B are such that $P(A) = 0.7$, $P(A \cap B) = 0.45$ and $P(A^1 \cap B^1) = 0.18$. Find;

(i) $P(B')$ (ii) $P(AorB)$

(b) M and N are two events such that $P(M) = 0.3$, $P(N) = 0.1$ and $P(M/N) = 0.2$, find:

(i) $P(M' \cup N')$, (ii) $P(M/N')$

(c) Events A, B and C are mutually exclusive and exhaustive such that $P(A) = k(1-2x), P(B) = k(1+x)$ and $P(C) = k(3+x)$. Find the value of

(i) k (ii) $P(B)$ if $x = \frac{1}{4}$.

(d) If two events C and D independent such that their chance of occurring together is $\frac{1}{5}$ and the chance that either C or D occurs is $\frac{7}{8}$.

(i) Show that C′ and D′ are also independent.

(ii) Find $P(\mathrm{C})$ and $P(\mathrm{D})$.

8. (a) A bag A contains 5 red ball and 3 black balls and bag B contains 3 red balls and a black ball. A bag is selected at random and two balls drawn from it without replacement. If A is twice as likely to be chosen as B,

(i) Find the probability that both balls are of different colours and the probability that the balls are from A given that they are of different colours.

(ii) Construct a probability distribution table for the number of black balls drawn. Hence find the mean number of black ball drawn and the variance.

(b) A box P contains 1 red, 3 green and 1 blue bead. Box Q contains 2 red, 1 green and 2 blue beads. A balanced die is thrown and if the throw shows a six, box P is chosen otherwise box Q is chosen. A bead is drawn at random from the chosen box. Given that a green bead is drawn, find the probability that it came from box P.

9. A random variable X has a p.d.f given by;

$f(x) = kx; \quad 0 \le x \le 3, \quad f(x) = 3k(4-x); \quad 3 \le x \le 4$

$f(x) = 0; \; x < 0 \; and \; x > 4.$

(a) Sketch $f(x)$ hence the value of k.

(b) Find $E(3X-1)$.

(c) Obtain the cumulative distribution, $P(X \le x)$. Hence find

(i) $P(X < 3.5)$, (ii) the median and inter quartile range.

10.(a) The continuous random variable Y is uniformly distributed in the interval $a < Y < b$. The lower quartile is 5 and the upper quartile is 9. Find

(i) the values of a and b, (ii) $P(\ 6 < X < 7)$

(iii) the cumulative distribution function F(x).

(b) Given that $X \sim R[32, 37]$, find;

(i) E(X)

(ii) the probability that X lies within one standard deviation of the mean.

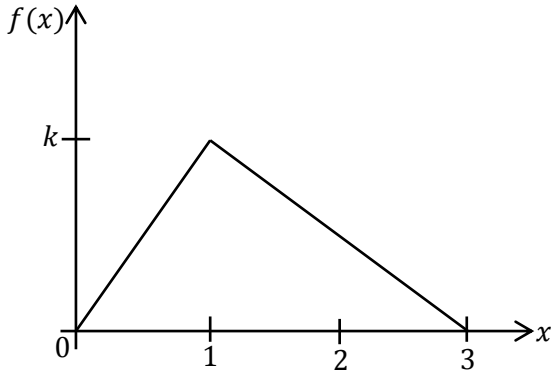
(c) A continuous random variable X has cumulative distribution function;

$F(\mathrm{X}) = \frac{x-2}{5},\ 2 \leq \mathrm{X} \leq 7$

Find

(i) $P(\mathrm{X} < 6/\mathrm{X} > 3)$ (ii) $E(\mathrm{X})$ and $Var(\mathrm{X})$,

11. A random variable X has its probability function as shown below.



(i) Find the value of k.

(ii) Find the probability density function, $f(x)$.

(iii) Compute $P(\ |X - 1| < \frac{1}{2})$

(iv) Derive the distribution function, F(x) and hence find the median of X.

12. (a) A couple is equally likely to produce a boy or a girl. Find the probability that in a family of five children there more boys than girls.

(b) A box contains equal number of red counters as yellow counters. A counter is taken from the box, its colour is noted the replaced. This is performed eight times in all. Calculate the probability that;

(i) exactly three will be red,

(ii) at least one will be red,

(iii) more than four will be yellow.

(c) A random variable X is such that $X \sim B(10, p)$ where $p < 0.5$ and $Var(X) = 1.875$. Find

(i) the value of p,

(ii) E(X),

(iii) $P(X = 2)$.

13. The masses of packets of sugar from a certain factory are normally distributed. In a large consignment of packets of sugar, it is found that 5% of them have a mass greater than 510 g and 2% have a mass greater than 515 g.

Estimate;

(a) the mean and standard deviation of this distribution,

(b) the probability that a packet picked at random from this consignment weighs more than 500 g.

14. The records from a Health Centre IV in Wakiso district showed that 80% of the patients who visited the centre on a certain day had malaria. Find the probability that on a particular day when 200 patients visited the centre;

(a) more than 70 patients tested positive for malaria,

(b) at least 55 patients were not suffering from malaria,

(c) less than 170 patients tested positive for malaria.

15. The heights of boys in a certain village follow a normal distribution with mean 150 cm and variance 25 cm^2. Find the probability that a boy picked at random from the village has height:

(a) less the 153 cm,

(b) more than 158 cm,

(c) between 149 cm and 159 cm,

(d) more than 10 cm difference from the mean height.

16. The masses of cows on Mr. Kato's farm of local cattle are normally distributed. It is discovered that 5% of the cows have a mass greater than 110 kg and 2% have a greater than 115 kg. Estimate the:
    (a) mean and standard deviation of this distribution,
    (b) number of cows in a group of 1000 cows which weigh between 109 kg and 121 kg.
    A veterinary doctor visited the farm and found that 10% of the cows malnourished. Determine the weight of the heaviest malnourished cow.

17.(a) The chance that a hen on Mrs. Musoke's poultry farm is infected with a deadly virus is 0.4. If a sample of 150 hens were inspected on the farm, find the 99.5% confidence limits for the mean number of cows that are infected.

(b) A 95% confidence interval for the mean life of a particular type of smartphone battery was calculated and the confidence limits were 1023.3 hours and 1101.7 hours. The interval was based on a sample of 36 smartphone batteries. Find the 99% confidence interval for the mean life of this type of batteries.

(c) A random variable of 50 readings taken from a normal population gave the following data: $\sum x = 163$ and $\sum x^2 = 548$. Calculate the:

(i) unbiased estimate for the population variance,
(ii) 98.46% confidence interval for the population mean.

MECHANICS

18.(a) ABC is an isosceles triangle, right angled at A with $\overline{AB} = 2$m. forces of 8N, 4N, and 6N act along the sides BA, CB, and CA respectively. Find the magnitude and direction of the resultant force.

(b) In a square ABCD, three forces of magnitude 4N, 10N, and 7N act along AB, AD and CA respectively. Their directions are in the order of the letters. Find the magnitude of the resultant force.

(c) In an equilateral triangle PQR, three forces of magnitude 5N, 10N, and 8N act along sides PQ, QR, and PR respectively. Their directions are in order of the letters. Find the magnitude of the resultant force.

19. A particle of mass 15kg is pulled up a smooth slope by a light inextensible string parallel to the slope. The slope is 10.5 m long and inclined at $\sin^{-1}(4/7)$ to the horizontal. The acceleration of the particle is $0.98ms^{-2}$.
Determine the:

(a) Tension in the string.

(b) Work done against gravity when the particle reaches the end of the slope

21. (a) A particle performs a SHM of periods 4s and amplitude $2cm$ about a centre O. find the time it takes the particle to travel from O to a point P, a distance $\sqrt{2}\ cm$ from O.

(b) A particle is moving with linear simple harmonic motion of amplitude 1.5m. The speed of the particle is $\sqrt{50}$ ms^{-1} when its displacement from the end point is 1m. Calculate its maximum acceleration.

(c) A particle moves with SHM about a mean position O. when passing through two points which are $2m$ and $2.4m$ from O the particle has speeds of $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the amplitude of the motion and the greatest speed attained by the particle.

22.(a) A body moving initially with a velocity u covers a distance x after t seconds. If it moves with a uniform acceleration a, derive an expression relating x, t, u, and a.

(b) A train approaching a station does two successive half kilometers in 16s and 20s respectively. Assuming a uniform retardation, calculate the further distance the train runs before it comes to rest.

(c) A body falls from rest from the top of a tower and during the last second it falls $\frac{9}{25}$ of the whole distance. Find the height of the tower.

23. A body moving with acceleration $e^{2t}\boldsymbol{i} - 3\sin 2t\boldsymbol{j} + 4\cos 2t\,\boldsymbol{k}$ is initially located at the point (1, -2, 2)m and has a velocity of $4\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ ms^{-1}. Find the;

a) Speed of the body when $t = \frac{\pi}{4}$s.

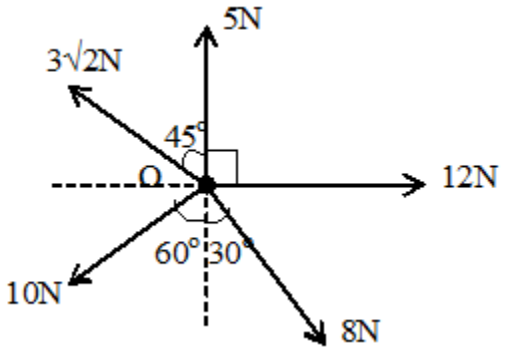
b) Distance of the body from the origin at $t = \frac{\pi}{4}$

24. (a) Forces of 7N and 4N act away from a common point and make an angle of θ^o with each other. Given that the magnitude of their resultant is 10.75N, find the;

(i) Value of θ^o

(ii) Direction of the resultant force

(b) In the diagram below find the magnitude and direction of the resultant force



25. A boy throws a stone at a vertical wall a distance, d away. Given that R is the maximum range on the horizontal through the point of projection that can be attained by the speed of projection, show that;

(a) the height above the point of projection of highest point on the wall he can hit is $\left(\frac{R^2-d^2}{2R}\right)$,

(b) in this case, the angle of projection is $tan^{-1}\left(\frac{R}{d}\right)$.

26. A ball is hit at a point O, which is at a height of 2m above the ground and at a horizontal distance 4m from the wall, the initial speed being in a direction of 45^0 above the horizontal. If the ball just clears the wall which is 1m high,

(a) show that the equation of path of the ball is $16y = 16x - 5x^2$.

(b) calculate the;

(i) distance from the net at which the ball strikes the ground.

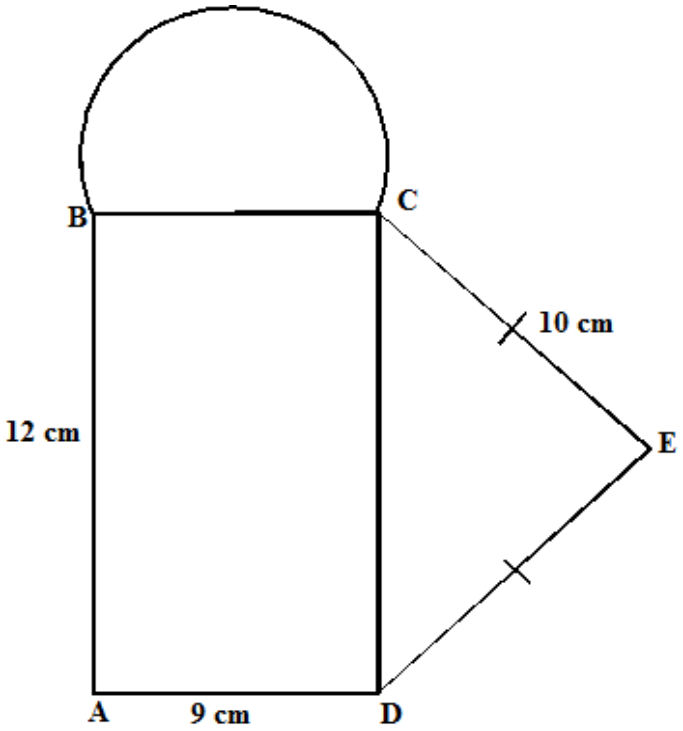
(ii) magnitude and direction of the velocity with which the ball strikes the ground.

27. With its engine working at a constant rate of $18k$W, a vehicle of mass 1.5 tones ascents a hill of 1 in 98 against a constant resistance to motion of $450N$. find;

(a) The acceleration of the vehicle up the hill when travelling with a speed of $10ms^{-1}$.

(b) The maximum speed of the vehicle up the hill.

28. The figure below represents a lamina formed by welding together rectangular, semi-circular and triangular metal sheets.



Find the position of the centre of gravity of the lamina from the sides AB and AD.

29. At 10:00 am, ship A and ship B are 16 km apart. Ship A is on a bearing N35^{o}E from ship B. ship A is travelling at 14 kmh^{-1} on bearing S29^{o}E. ship B is travelling at 17 kmh^{-1} on a bearing N50^{o}E. determine the;

(a) Velocity of ship B relative to ship A

(b) Closest distance between the two ships and the time when it occurs

30. (a) Two ships A and B are observed from a coast guard station and have the following displacement velocities and times.

Ship	Displacement	Velocity	Time (t)
A	$(\boldsymbol{i}+3\boldsymbol{j})$ km	$(\boldsymbol{i}+2\boldsymbol{j})kmhr^{-1}$	12:00 hours
B	$(\boldsymbol{i}+2\boldsymbol{j})$ km	$(5\boldsymbol{i}+6\boldsymbol{j})kmhr^{-1}$	13:00 hours

Find the time when the two are closest to each other.

(b) If at 13:00 hours ship A changed it's velocity to $\left(\frac{11}{3}\boldsymbol{i}+2\boldsymbol{j}\right)kmhr^{-1}$, show that they collide and find the time and position of collision.

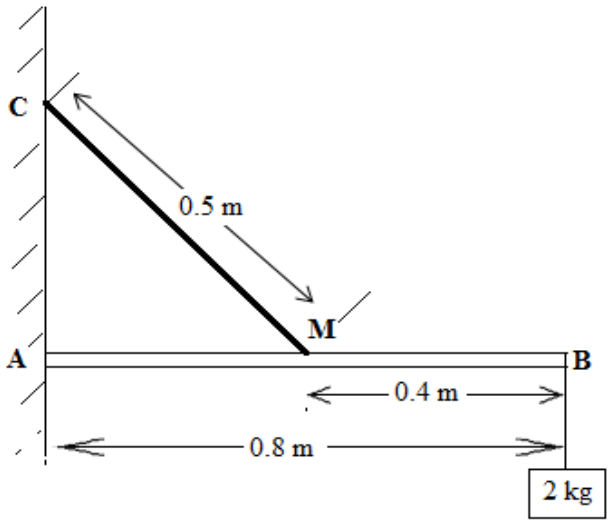
31. A light inextensible string has one end attached to a ceiling, the string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley, the particle of mass 5 kg is attached at the free end of the string, the sections of the strings not in contact with the pulleys are vertical, if the system is released from rest and moves in a vertical plane, determine the;

(a) Accelerations of the 2kg and 5kg masses

(b) Tensions of the 2kg and 5kg masses

(c) Distance moved by the system in 1.5 seconds.

32. The figure below shows a uniform beam of length 0.8 metres and mass 1 kg. The beam is hinged at A and has a load of mass 2 kg attached at B.



The beam is held in a horizontal position by a light inextensible string of length 0.5 metres. The string joins the mid-point M of the beam to a point C vertically above A. Find the:

(a) Tension in the string.

(b) Magnitude and direction of the force exerted by the hinge.

33. A non-uniform ladder AB of length 10m, weighing $5W$ and centre of mass $4m$ from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. The angle between the ladder and the horizontal is 50^{o} and the coefficient of friction at each end is $\frac{1}{4}$ and $\frac{1}{2}$ respectively.

(a) A man of weight $13W$ begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?

(b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

34. A particle of mass 2 kg is acted upon by a force of 21N in the direction $2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$. Find in vector form the;

(a) Force

(b) Acceleration hence its magnitude.

35. Five forces of magnitudes 3N, 4N, 4N, 3N and 5N act along the lines AB, BC, CD, DA, and AC respectively, of a square ABCD of side 1m. The direction of the forces is given by the order of the letters. Taking AB and AD as reference axes; find the

(a) Magnitude and direction of the resultant force.

(b) Equation of the line of action of the resultant force and hence find the point where the resultant force cuts the side AB.

36. A particle of mass 4 kg starts from rest at a point $(2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ m. it moves with acceleration $\boldsymbol{a} = (4\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})ms^{-2}$ when a constant force $\boldsymbol{F}$ acts on it. Find the:

(a) Force $\boldsymbol{F}$.

(b) velocity at any time t.

(c) work done by the force **F** after 6 seconds

35. At **10.00am**, ship **A** moving at **20km/hr** due east is **10km** South East of another ship***B***. If ***B*** is moving at **14km/hr** in direction **S30^{0}W** and the ships maintain their velocities, find the;
   (a) Time when the ships are **closest** together and the **shortest distance** between the ships.
   (b) Bearing of **A** from ***B*** at that time.

36. A particle A initially at the point with position vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ km is moving with a constant velocity of $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ kmh^{-1}. At the same instant, a particle B at the point $(3,3,2)$ is moving with a constant velocity of $3\mathbf{i} - 2\mathbf{k}$ kmh^{-1}. Find the:
   (i) relative velocity of particle A to B.
   (ii) relative displacement of particle A to particle B at any instant.
   (iii) shortest distance between the two particles in their subsequent motion.

37. (a) Two ships A and B are observed from a coast guard station and have the following displacement velocities and times.

Ship	Displacement	Velocity	Time (t)
A	$(\boldsymbol{i} + 3\boldsymbol{j})$ km	$(\boldsymbol{i} + 2\boldsymbol{j})kmhr^{-1}$	12:00 hours
B	$(\boldsymbol{i} + 2\boldsymbol{j})$ km	$(5\boldsymbol{i} + 6\boldsymbol{j})kmhr^{-1}$	13:00 hours

Find the time when the two are closest to each other.

(b) If at 13:00 hours ship A changed it's velocity to $\left(\frac{11}{3}\boldsymbol{i} + 2\boldsymbol{j}\right)kmhr^{-1}$, show that they collide and find the time and position of collision.

NUMERICAL METHODS

38. (a) Show that the iterative formula based on Newton Raphson's method for solving the equation $e^{2x} + 4x = 5$ is given by,

$$x_{n+1} = \frac{e^{2x_n}(2x_n - 1) + 5}{2e^{2x_n} + 4}, \text{ n = 0, 2, 3...}$$

(b) (i) Construct a flow chart that;

- reads the initial approximation x_0,
- computes, using the iterative formula in (a) and prints the root of the equation $e^{2x} + 4x - 5 = 0$, and the number of iterations when the error is less than 1.0 x 10^{-4}.

(ii) Perform a dry run of the flow chart when $x_0 = 0.5$.

39. The iterative formulae below are used for calculating the positive root of the $f(x) = 0$.

$$\text{A: } x_{n+1} = \frac{1}{3}\left(\frac{2x_n{}^3+12}{x_n{}^2}\right)$$

$$\text{B: } x_{n+1} = \sqrt{\left(\frac{x_n{}^3+12}{2x_n}\right)}$$

(a) Taking $x = 2$, use each formula twice and hence deduce the most suitable for solving $f(x) = 0$.

(b) Find the root of the equation $f(x) = 0$, correct to three decimal places.

(c) Find the equation whose root is in b) above.

40. (a) Show that the equation $\ln x = \sin x + 2$ has a root between $x = 3$ and $x = 4$. Use linear interpolation to estimate the initial approximation x_o to 1 decimal place.

(b) Using the x_o above and the Newton Raphson method find the root correct it 3 decimal places.

41. (a) The table below shows the values of x and their corresponding natural logarithm

x	5.0	5.2	5.4	5.7	6.0
Inx	1.609	1.647	1.686	1.740	1.792

Use linear interpolation or extrapolation to find

(i) $In\ (5.56)$,

(ii) $e^{1.575}$.

(b) A car consumed fuel amounting to shs 14,800, shs 15,600, shs 16,400 and shs 17,200 in covering distances of 10km, 20km, 30km and 40km respectively. Estimate the;

(i) cost of fuel consumed for a distance of 45km,

(ii) distance travelled if fuel of shs 16,000 is used.

42. (a) Two positive decimal numbers X and Y were approximated with errors E_1 and E_2 respectively. Show that the maximum possible relative error in the approximation of the product X^3Y^2 is $3\left|\frac{E_1}{X}\right| + 2\left|\frac{E_2}{Y}\right|$.

(b) Given that X = 5.64 and Y = 10.0, rounded off to the given number of decimal places, find the;

(i) maximum possible errors in X and Y,

(ii) percentage error made in the approximation of X^3Y^2.

43.(a) Given that $y = xsinx$ and $x = 2$, find the absolute error in y giving your answer correct to 3 significant figures.

(b) The numbers $x = 1.5, y = -2.85$ and $z = 10.345$ were all rounded off to the given number of decimal places. Find the range within which the exact value of $\frac{1}{x} - \frac{1}{y} + \frac{y}{xz}$ lies.

44. (a) Using trapezium rule with five strips evaluate $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}\,dx$, correct to three decimal places.

(b) Find the exact value of $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}\,dx$.

(c) Find the percentage error in the approximation in a) above and suggest how this error can be reduced.

END